

## Mirror Deformation from mirror/spacer differential thermal expansion and its contribution to the cavity Zero Crossing Temperature

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ULE is manufactured in a meter-diameter disk, and due to inhomogeneity in the relative concentration of  $TiO_2$  in the  $SiO_2$ , the material in the disk has a corresponding inhomogeneity in its coefficient of thermal expansion. The thermal expansion, which is temperature dependent, is of the form

$$\alpha(T) = K_0 + 2.21 T - 0.0112 T^2 \text{ ppb/C}$$

and it is the constant  $K_0$  which is variable and sets the temperature at which the expansion coefficient crosses zero [1]. A linear estimate of the expansion coefficient change can be seen to be 2 ppb/C.

It is possible to measure the Zero Crossing Temperature of the ULE from a measurement of the speed of sound in the material [2]. The reputation of this calibration has not always been excellent, but new measurements [3] have produced reliable and accurate results [3], and this method is being refined by Richard Fox at NIST, Boulder. The measurements have an accuracy of about ±1 C, and this comes about equally from the present knowledge of the relation between the speed of sound and thermal expansion, as well as experimental error.

While the spacer material can be characterized for Zero Crossing (ZC) temperature with a speed of sound measurement, the best operating temperature for the cavity can be different as the mirrors are usually not made from the same material as the spacer. We discuss here that the difference is likely to be about one or two degrees C.





A finite element model of an ATF 6020-4 notched 50 mm diameter cavity (with a mirror) gives a calculation of the deformation of the mirror due to differential thermal expansion. From the diagram, the mechanism is that the mirror bends outward, as the mirror material expands more than the spacer material, but the mirror is pinned by the optical contact. We plot below the deformation from one mirror due to a thermal expansion difference of 10 ppb (which corresponds to a ZC difference of ~5 C).



This calculated difference gives an end-to-end cavity length change of 8  $10^{-11}$  m for a 1 C temperature change, or an effective thermal expansion coefficient of ~  $10^{-9}$  K<sup>-1</sup> for a cavity length of 0.1 m. With the temperature dependence of the spacer material being 2 ppb/C, the zero crossing of the cavity will likely differ from the zero crossing of the spacer alone by ~ 1 C. This difference is of the same scale as the error in the zero crossing measurement. For the mirrors having a greater expansion coefficient than the spacer, the cavity ZC is moved to a lower temperature.

For a midplane cavity (ATF 6030), the mirror expansion contributes similarly to the cavity expansion. This is shown graphically below in Figure 3. This plot come from Richard Fox (NIST, Boulder), who did the calculations and made the graph.



Figure 3. The cavity ZC temperature  $T_c$  is read off on the sloped line, for a given spacer zero crossing temperature (upper axis) and mirror ZC (right axis). For example, a spacer with ZC of 20 C and mirrors with ZC ~ 50 C give a cavity  $T_c$  of 25 C. ULE is specified to have a ZC between 3 and 35 C.

In summary, both the notched cavity and midplane cavities can have their cavity ZC temperature shifted from the spacer ZC temperature, and the coefficient is about 1 degree of shift for 5 degrees of ZC difference. An average shift could be expected to be a degree. An added note is that coating differential thermal expansion has an effect that is a small fraction of one degree.

## References

[1] Richard W. Fox, *Temperature analysis of low-expansion Fabry-Perot cavities*, Optics Express **17**, p15023 (1999).

[2] H.E. Hagy and W.D. Shirkey, *Determining absolute thermal expansion of titania-silica glasses: a refined ultrasonic method,* Applied Optics **14** p2099 (1975)

[3] R. W. Fox, *Effect of Structural Distortion on Fabry-Perot Temperature Response"* in *Conference on Lasers and Electro-Optics/International Quantum Electronics Conference, OSA Technical Digest (CD)* (Optical Society of America, 2009), paper CWI3